MEM 733 Applied Optimal Control I, Project

Professor Harry Kwatny

Wheeled Robot

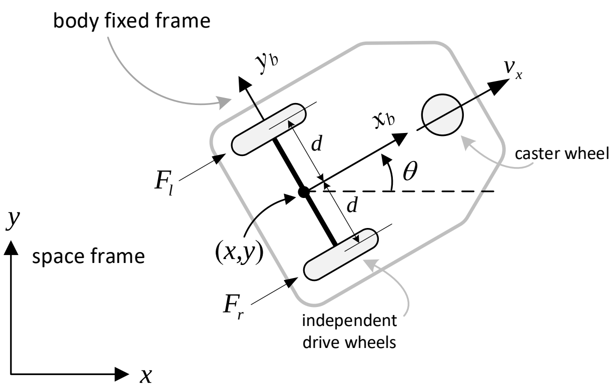

Figure 1: A simple wheeled robot. Drive and steering are accomplished by coordinating the angular velocities of the two drive wheels. The third wheel is a castor wheel and simply provides support.

The simplest wheeled robots are similar to the one shown in Figure 1. Even though its mathematical model is simple in structure, it is the prototype of a 'nonholonomic' dynamical system and presents difficult control properties. The kinematic model of this system, often used in control system analysis is

$$\frac{d}{dt}\begin{bmatrix} x \\ y \\ \theta \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} v \\ \omega \end{bmatrix} \tag{1}$$

The variables x, y denote the space location of the origin of a body fixed reference frame with the body x -axis pointing forward along the robot centerline and the body y -axis pointing left along the axle. θ denotes the angular orientation of the body x -axis in the space frame. The velocity v always points along the body x -axis and the vehicle angular velocity ω is positive in the clockwise direction. v and ω are considered independent controls with $u_1 = v \in [0,1]$ and $u_2 = \omega \in [-1,1]$. The goal is to steer the robot from an initial point $x = 5, y = 1, \theta = 0$ to the origin $x = 0, y = 0, \theta = 0$ in a way that minimizes

$$J = \int_0^{t_f} \left(1 + |u_1| + |u_2|\right) dt \tag{2}$$

Determine the optimal control and state trajectories.

Repeat the problem, but this time with an obstacle defined by

$$(x-2)^2 + y^2 - (1.5)^2 \le 0 \tag{3}$$